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Separation of Regional and Residual Components by Finite Element Analysis – A New Approach for Analysis of Water Level Data

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Abstract. Trend surfaces are generally used in the study of water level data to understanding the causes and effects of various trend surfaces. In the present paper the separation of regional and residual components of water level data is attempted using a method based on the Finite Element Analysis techniques. The residual is obtained by calculating the difference between the computed value of the trend surface at a point and the value of observed actual surface at that point. If the trend surface is thought to be regional or large scale component representing the total aquifer then the residual value can be considered the local ore small scale component representing the local variations in the aquifer. Removal of the regional trend has the effect of isolating and emphasizing local components represented by the residual values. Various techniques have been proposed and are widely in use for the separation of regional and the residual components, specially for separating the geophysical data. But the main drawback of all these techniques is that the regional component, so computed, has always the remnance of the residual components. Hence, the regional and residual components do not give a clear picture of the variations. In the present paper a new technique is suggested, in which the regional and residual components are computed using finite element analysis technique. This technique requires the water level data at only eight or twelve points representing the aquifer boundaries for the computation of regional component. A case history is presented wherein the data from the literature is analyzed using the technique proposed. The paper gives the details of the method and its advantages over the other methods which are supported by its application on the field data.

Introduction

One of the most common measurements in ground water investigations is the determination of the depth to ground water level. It (in the form of water table in unconfined aquifers and the piezometric surfaces for confined aquifers) indicates the elevations of atmospheric pressures of the aquifer. Water level data is generally analyzed for time variations of these levels. Secular variations, seasonal variations and short term variations are analyzed for understanding the changes and hence, the behavior of the aquifer system. Fluctuations due to tides, evapotranspiration etc., are also studied.

Water level data provide records of short term changes and long term trends of fluctuation of storage within ground water reservoirs. Recognized uses of ground water levels include: To identify the areas of detrimentally low or high water level, facilitate prediction of ground water supply out look for the future by showing the time – rate of change in ground water storage, to provide information for evaluating water yielding properties of ground water, to appraise the relationships between water level fluctuation and pumping, precipitation and other factors, to indicate the status of ground water in transit, to aid estimating the base flow in streams, to furnish information for use in research. Water level in artesian wells are affected by recharge in precipitation in the water table portion of the aquifer, withdrawals from wells and springs, earth tides changes in atmosphere pressure, surface water storage and surface loading. Water levels in wells are constantly fluctuating and decline or rise within relatively short time. Water level in water table aquifer change by direct recharge from precipitation, evaopotranspiration, withdrawals from wells, discharge to streams and sometimes a change in atmospheric pressure. Fluctuations in water level indicate change in (a) actual quality of water stored in aquifer and (b) movement of ground water.

The analysis of the piezometric levels and water levels distribution and their variation with time constitute an inexpensive, while very accurate, method for the behaviour analysis of an aquifer. The interpretation of the piezometric surface, besides permitting computations of some hydrodynamic parameters, leads to important conclusions concerning the hydrogeological characteristics of the aquifer and the ratio recharge discharge.

Piezometric surfaces at various times for the same aquifer give the flow direction which along with the transmissibility and the storage coefficient data can be interpreted in more meaningful manner.

Trend analysis deals with the recognition, isolation and measurement of trends that can be represented by lines, surfaces etc. As generally used for geological data, the trend surface analysis has invariably involved the filling of trend surfaces to satisfy the least squares criterion. This results in the trend surfaces passing through above or below each actual data point. The difference between the computed value of the trend surface at the point and the value of observed actual surface at that point is termed as the residual value. In satisfying the least squares

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criterion the sum of squared residuals is minimized. Hence, if the trend surface is thought as the regional (large scale component) then the residual value can be considered the local (small scale component). Residual is influenced by and hence may represent: Change in lithology, changes in transmissibility and storage coefficient and structures present in the area.

Method of Computing Residual

Computing residuals involves two steps, first: predicting the values expected from deep/ regional features and second: subtracting them from observed values so as to leave the effect of shallow / local features. In practice, the methods available determine the expected value of the regional by averaging the values in the area surrounding the station 4 and hence the separation is never complete and both regional and residual are distorted by the effects of each other.

In the method presented and discussed below this lacunae is removed as the regional are computed by taking into account the values at the nodes of the selected area where the effect of local features is presumed to be absent. The present approach utilizes the concept and properties of the basis or shape functions (Zienkiewicz 1979; Cheung and Yeo 1979) used in finite element analysis. This study takes advantage of very useful interpolating properties of the isoparametric elements and not dealing with the solution of any differential equation, as one is accustomed to by finite element analysis (Mallick and Sarma 1992; Karkhanis *et al* 1997). In this study the methodology with the help of Fig 1(a) and 1(b) has been explained. The x-y reference space is identified by the nodes represented by filled circles and numbered 1 through 8. The field values (of water level) at these nodes lying on the peripherals are assumed to represent the regional. With simple substitutions $\xi = (xxc)/a$ and $\eta = (y-yc)/b$ where xc and yc define the centre of the element and 2a and 2b are the sides of the rectangle. Figure 1 (b) represents the real space and the element representing the field data of water level can be represented by a reference element in Fig. 1(a) defined by non-dimensional coordinates ξ and η varying between –1 and 1. The element can be a square, a rectangle or a quadrilateral. We have chosen a square. The nodes 1 through 8 in reference element correspond to the nodes of the element in x-y space.

The field variable, the water level data in this case, at any point (ξ,η) in the reference element can be expressed by a series, which in fact is a weighted sum of the nodal water level observation values P(say) $P(\xi,\eta) = \Sigma Ni(\xi,\eta).$ Pi $i = 1,2,3,\ldots..8.$ (1) Where the weights, $Ni(\xi,\eta)$ are the shape functions of the elements. Equation 1 refers to a quadratic element and the shape functions are given by:

$Ni(\xi,\eta) = (1+\xi\xi i)(1+\eta\eta i) * (\xi\xi i + \eta\eta i - 1)/4$ $i = 1,3,5,7$
$Ni(\xi,\eta) = (1-\xi2)(1+\eta\eta i)/2$ $i = 2,6$
$Ni(\xi,\eta) = (1+\xi\xi i)(1+\eta2)/2$ $i = 4,8$ (2)

Where ξi and ηi are the nodal coordinates. It may be noted that $Ni(\xi,\eta) = 1$ at *i*th node and zero elsewhere. Furthermore, $\Sigma Ni(\xi,\eta) = 1$. Cheung and Yeo (1979) have described other properties of the shape functions and their expressions for cubic and other elements.

The regional anomalies computed by equation 1 in the reference element need to be translated to the real map space. This is carried out by a geometric transformation similar to equation 1.

$x(\xi,\eta) = \Sigma Mi(\xi,\eta).\ xi$
$y(\xi,\eta) = \Sigma Mi(\xi,\eta).\ Yi$ (3)

In case of isoparametric elements used here, $Mi(\xi,\eta) = Ni(\xi,\eta)$. xi and yi are the nodal coordinates shown in Fig. 1(b). With this brief account on finite element approximation, we proceed to illustrate its application to two field cases.

Field Example: 1

In 1964, Saskatchewan Research Council conducted extensive test hole programme aimed at delineating the sand and gravel aquifers in the buried Missouri river and Yellowstone river valleys west and north west and north of Estevan in SE Saskatchewan, Canada. According to Meneley *et al.* (1957) the valley of preglacial Missouri river can be traced from North Dakota in to SE Saskatchewan. The preglacial Missouri river was joined by preglacial Yellowstone river at a point 15 miles north of international border. Based on the investigations of the sand and gravel deposits in the buried Missouri and Yellowstone river valley, subsequent resistivity studies and test hole results, it was suggested that the deepest parts of the channels of the preglacial Missouri and Yellowstone river occur at the lower altitudes as anticipated by Meneley *et al.* (1957). A permeable sand and aquifer in the lower part of the drift was encountered in the test holes.

Figure 2 (a) gives the study area and its surroundings. Water level contour maps prepared from the data for wells terminating in the bed rock or glacial drift at depths between 150-250 feet (Walton, 1970) are shown in Fig. 2 (b). Valleys in the piezometric surface correspond with the Souris river and its tributaries. Contours are also warped to some extent in the vicinity of the buried valley system especially north and west of Estevan. Fig. 2 (c) is a water level contour map prepared from the data for wells terminating in the bedrock or glacial drifts at depths

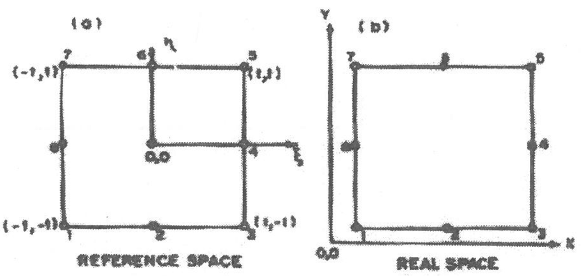


Fig. 1 (a) reference space (b) real space.

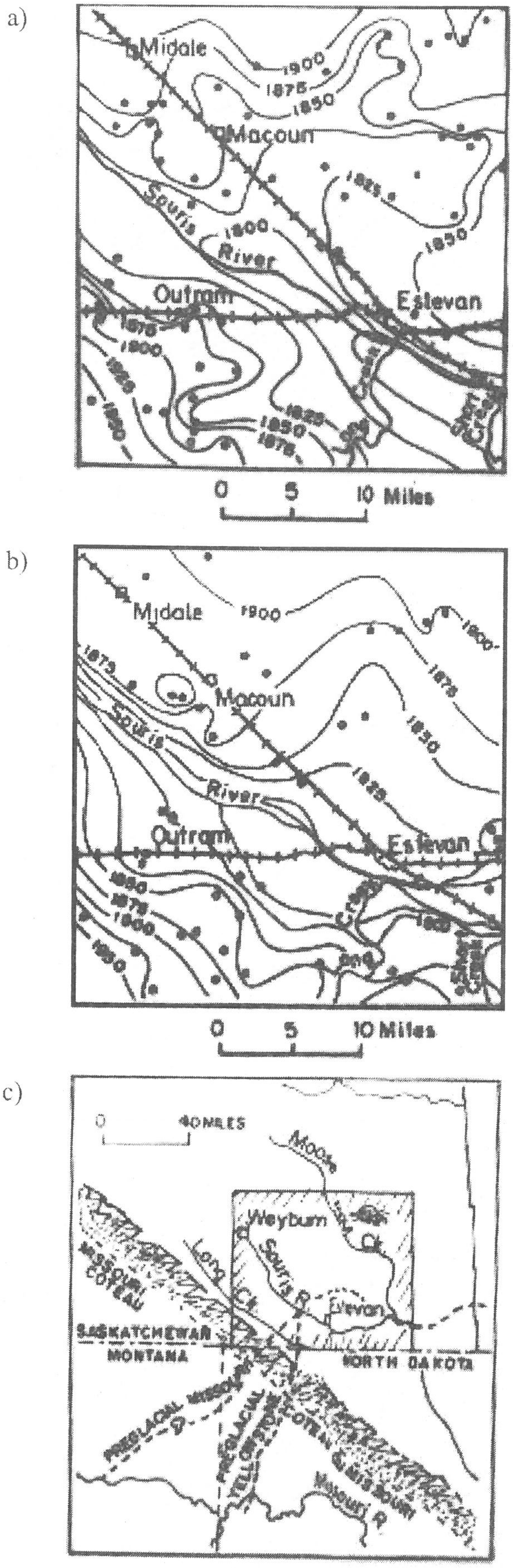
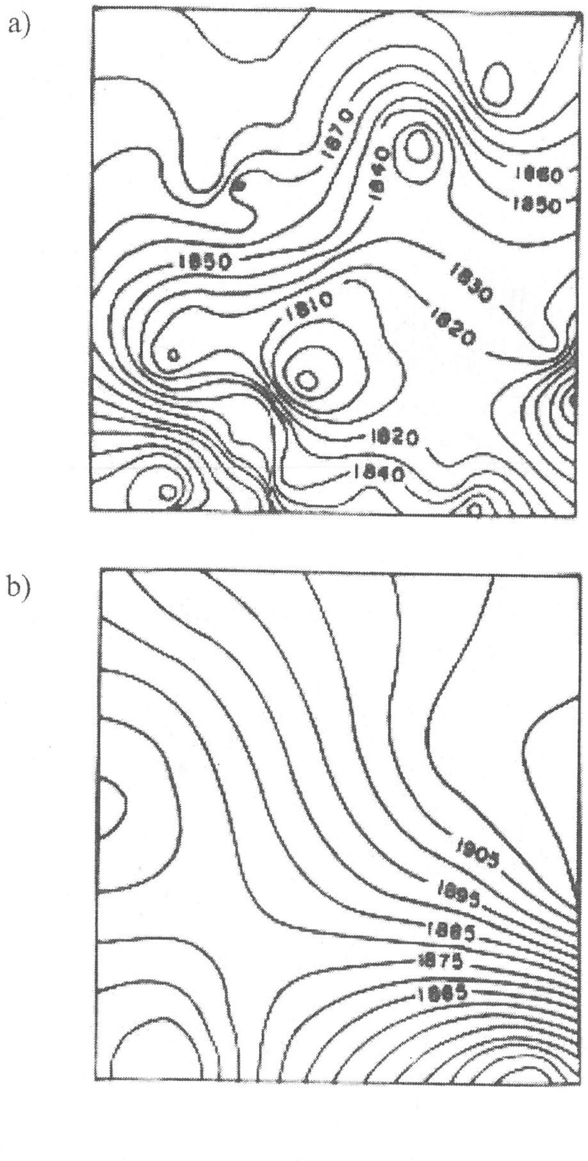


Fig. 2 (a) Location map of the study area (b) Piezometric levels for the shallower aquifers (150-250 feet) and (c) Piezometric levels for deeper aquifers (250-450 feet).

between 250-450 feet. Besides similarity of this map with the one for depth 150-250 feet, the large and deep valley in the 7 piezometric surface over the buried valley suggests that permeable sand and gravel deposits of large areal extent and thickness exist at depths extending 250 feet.

The proposed method was utilized for the analysis of the piezometric data from this area. The piezometric levels were digitized at the well locations and maps for the two aquifers were prepared and are shown in Fig. 3 (a) and Fig. 4 (a) for the shallow and deep aquifers respectively. Figure 3 (b) and Fig. 4 (b) give the regional components as separated using the proposed method where as Fig. 3 (c) and Fig. 4 (c) give the picture of the residual piezometric levels respectively. It must be seen that the regional behavior is more or less same for the two sets of aquifers. When we compare the behavior of the residuals we find that it does follow the similar pattern as that of the total level, with some marked differences specially with respect to the deeper aquifer. There appears new closures in the middle and the southern portion of the contour map of the residuals. The images prepared using these data bring out variations very clearly.



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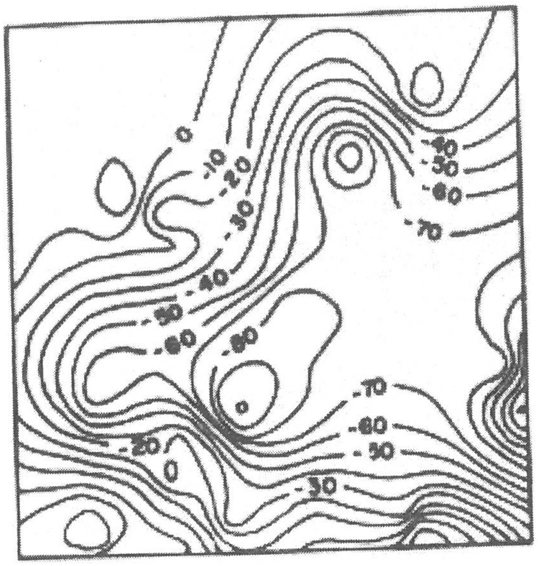
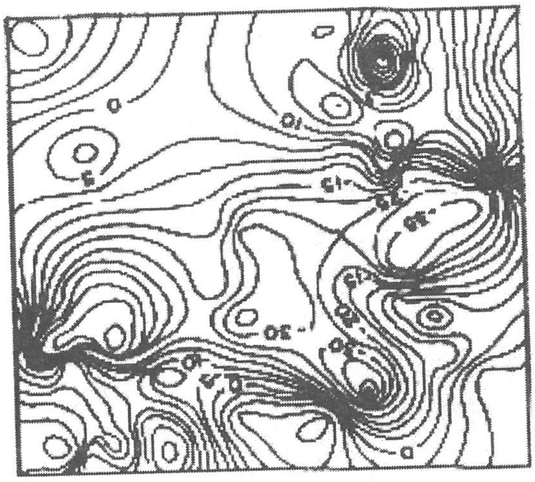
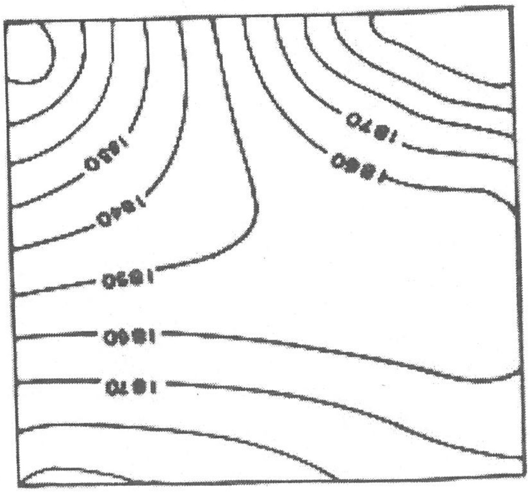


Fig. 3 Behavior of the shallow aquifer (a) total piezometric level (b) its regional component and (c) its residual component.

a)



b)



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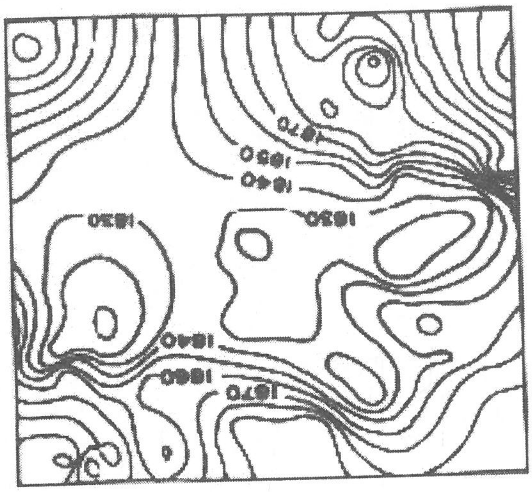
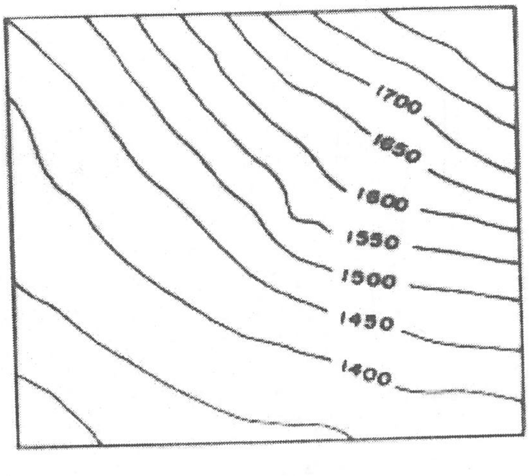
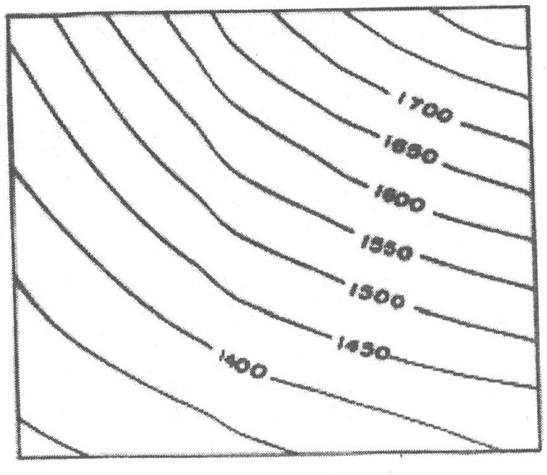


Fig. 4 Behavior of the deeper aquifer (a) total piezometric level (b) its regional component and (c) its residual component.

a)



b)



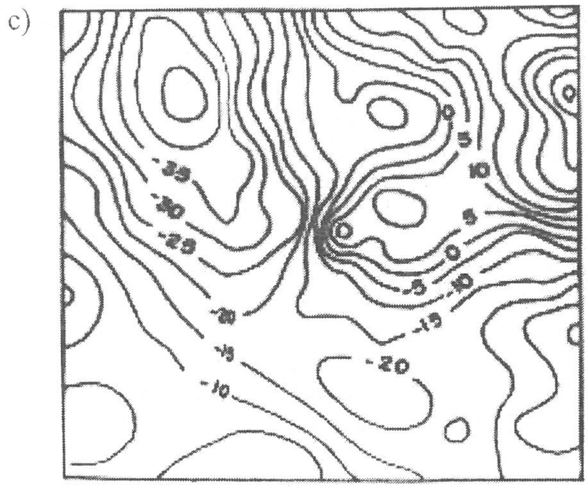


Fig. 5 Ground water table map (a) its regional (b) its residual (c) for the year 1950.

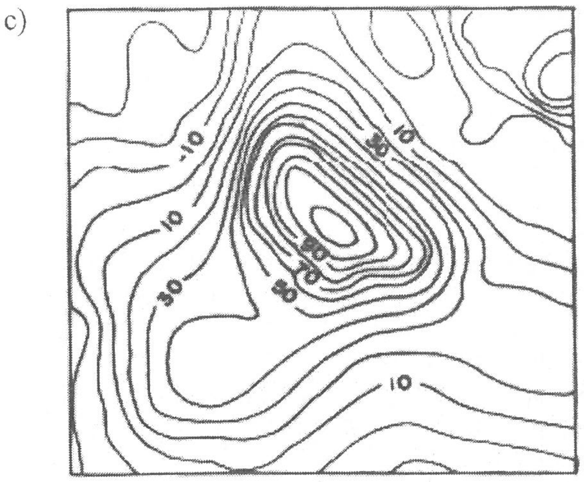
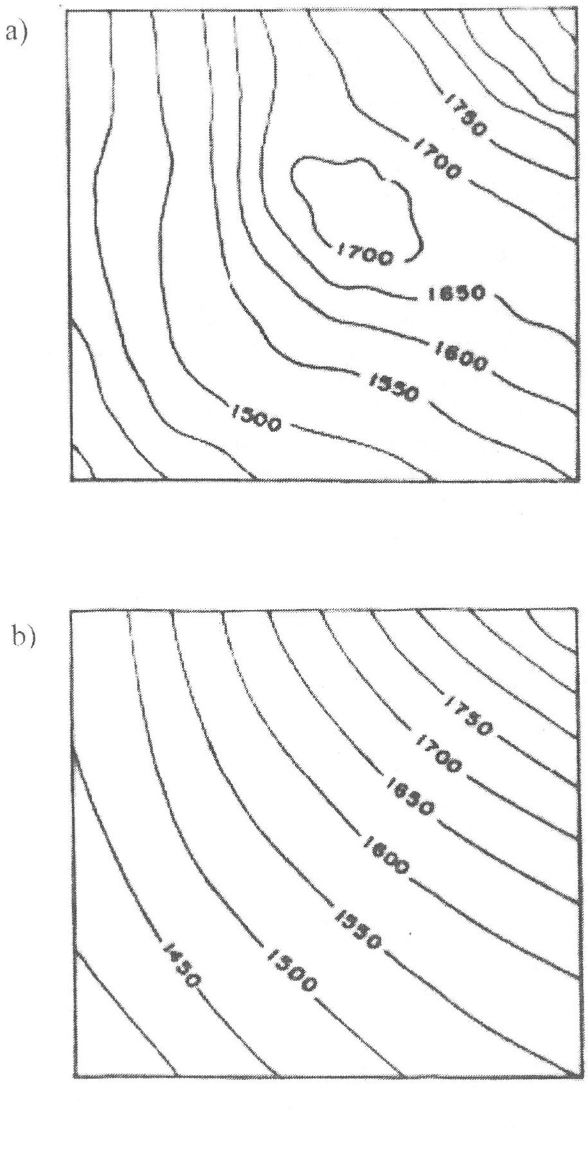


Fig. 6 Ground water table map (a) its regional (b) its residual (c) for the year 1960.



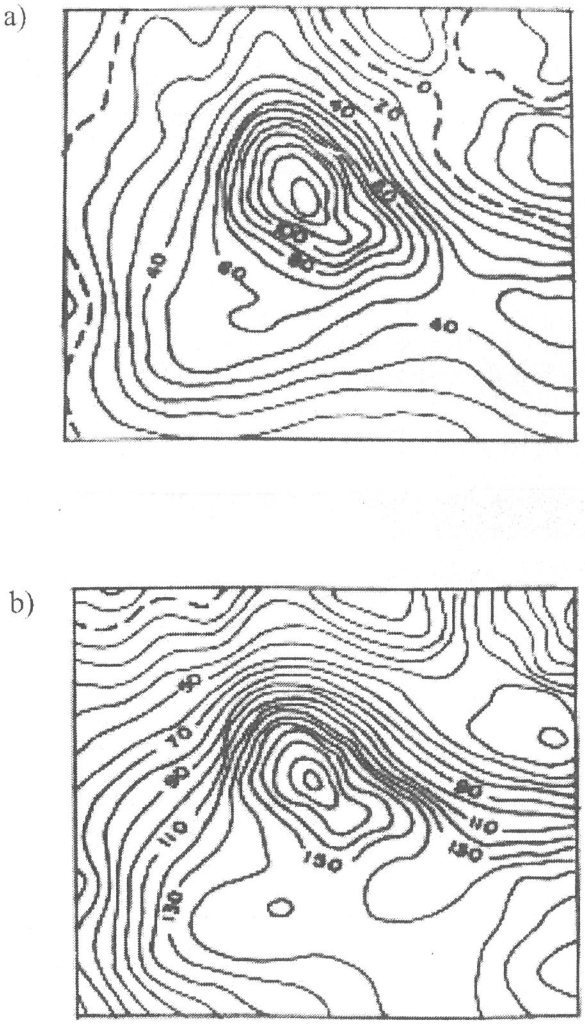


Fig. 7 Difference map is constructed by subtracting the two matrices of the grid values (a) & (b).

Field Example: 2

Davis (1986) while discussing the similarity of map takes up an example of ground water table (above mean sea level in feet) from an area in Nebraska. Map shown in Fig. 5 (a) was prepared from the data gathered in 1950 and shows ground water table undisturbed except for the effect of local pumping. Map shown in Fig. 6 (a) was prepared from the data collected ten years later after the construction of a large dam near the middle of the area. The contribution of the reservoir to the ground water levels has been significant and has been shown by the difference map (Fig. 7 b). These maps have been prepared using an automatic contouring programme that constructs an intermediate grid. The difference map is constructed by subtracting the two matrices of the grid values from each other and then contouring the resulting matrix. In this area the observation wells used to get the two sets of data are not the same.

For the present study the maps in Fig. 5 (a) and Fig. 6 (a) are taken and digitized at the well locations. These values are taken to process the data using the proposed method. Figure 5 and 6 give the ground water table map (a), its regional (b) and its residual (c), for the year 1950 and 1960 respectively. Another set of map is prepared using the difference of maps given in Fig. 5 (c) and 6 (c) giving the effect of the dam on the residual of the area.

The following observations can be made:

- There is a general increase in water level from SW to NE as seen in the regional maps of 1950 and1960.

- There are zones of low water level in the west and south of the area and high in the Northeast area in 1950 as seen in the residual map. Whereas after the construction of dam these zones improved and a high is observed at the central part of the study area.

- It is seen that the difference map using the raw data gives a very high rise in the water table because of the construction of dam whereas the actual effect of dam is only ~130 ft. and not 200 ft. as being made out. The rest is only due to the regional change in the water level.

- The method suggested has been useful in separating the regional and the residual effects (as seen in the preceding section) as the form of changes in the water level due to construction of the dam.

Conclusion

The paper presents a new methodology to separate the regional and residuals of the water level and piezometric level data. The method has been explained and tested on two field examples for its efficacy and use. The proposed method overcomes the difficulties of the earlier methods by making use of the data only at the nodal points for computing the regional component. The results show that the method is capable of separating various components of the water table data and hence provides a powerful tool to interpret the data.

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