

Comparison of Weibull and Gaussian Mixture Models for Wind Speed Data Analysis

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Abstract: In order to have a reliable estimate of wind energy potential of a site, high frequency wind speed and direction data recorded for an extended period of time is required. Weibull distribution function is commonly used to approximate the recorded data distribution for estimation of wind energy. In the present study a comparison of Weibull function and Gaussian mixture model (GMM) as theoretical functions are used. The data set used for the study consists of hourly wind speeds and wind directions of 54 years duration recorded at Ijmuiden wind site located in north of Holland. The entire hourly data set of 54 years is reduced to 12 sets of hourly averaged data corresponding to 12 months. Authenticity of data is assessed by computing descriptive statistics on the entire data set without average and on monthly 12 data sets. Additionally, descriptive statistics show that wind speeds are positively skewed and most of the wind data points are observed to be blowing in south-west direction. Cumulative distribution and probability density function for all data sets are determined for both Weibull function and GMM. Wind power densities on monthly as well as for the entire set are determined from both models using probability density functions of Weibull function and GMM. In order to assess the goodness-of-fit of the fitted Weibull function and GMM, coefficient of determination (R^2) and Kolmogorov-Smirnov (K-S) tests are also determined. Although R^2 test values for Weibull function are much closer to '1' compared to its values for GMM. Nevertheless, overall performance of GMM is superior to Weibull function in terms of estimated wind power densities using GMM which are in good agreement with the power densities estimated using wind data for the same duration. It is reported that wind power densities for the entire wind data set are 307 W/m² and 403.96 W/m² estimated using GMM and Weibull function, respectively.

Keywords: Ijmuiden, Gaussian mixture model, Weibull, wind speed, wind rose.

Introduction

Theoretical modeling of wind speed and prediction of stored wind energy potential for a site requires comprehensive and accurate description of measured wind speed and wind direction data. Additionally, reliable fitting of measured wind speed data to a theoretical function and subsequent energy generation is sensitive to transient processes and rapid fluctuations present in wind. A suitable theoretical function is selected for modeling such fluctuations in measured data. This function gives an accurate assessment of wind potential of the site under investigation thereby selecting a suitable wind turbine for the site. Compliance of these factors leads to a realistic and accurate measure of wind energy.

Various continuous mathematical functions are used to represent measured wind data distributions. Commonly used distribution functions in different studies include Weibull, Rayleigh, bimodal Weibull, lognormal and gamma functions. In majority of studies Weibull function with two parameters namely shape (k) and scale (c) is used, stems from

the fact that Weibull function gives flexibility over the range of parameters values. Weibull parameters k and c describe the breadth and abscissa of the measured distribution, respectively. Chang (2011) used Weibull function to determine wind potential for three different sites in Taiwan. The study revealed that MLM, MMLM and MoM are better estimation methods for Weibull parameters. Carta et al. (2008) approximated recorded wind speed distribution by various probability functions. The study showed that best results are obtained from Weibull probability function. Zhou et al (2006) conducted the analysis of measured wind speeds using Weibull as a modeled function for Pearl river delta region near south China sea and Hong Kong. Using the fitted model authors also calculated wind potentials for both sites. Seguro and Lambert (2000) used Weibull formula for modeling wind data and calculated k and c parameters using MMLM.

Although Weibull function is quite sufficient in describing wide ranging wind speed values but is ill-modeled for data distribution showing bimodal behavior. In such situations, hybrid distribution functions are used to estimate wind energy potential

and are quite accurate in describing wind characteristics of the sites under investigation. These hybrid wind models are represented by a combination of bimodal and Weibull function.

Materials and Methods

Weibull Distribution Function

Wind fields and transfer of heat energy from one part of the earth to another are dependent upon the solar distribution in the earth's atmosphere. The associated motion of air particles corresponds to large amount of kinetic energy that can be harnessed and utilized for the benefits of mankind. Power density varies as a cube of actual measured wind speeds and is given by:

$$P_A = \frac{1}{2} \rho v^3 \quad (1)$$

where $\rho = 1.225 \text{ kg/m}^3$ density of air, P_A actual wind power density (W/m^2) from measured wind speeds, ' v ' (m/s). Wind speeds are fitted to a continuous function to obtain *pdf* and power density. Wind power density estimated using fitted functions results in a lowering of uncertainty estimated power density. Wind speed data recorded on long term basis is used to construct a probability density function (*pdf*). Commonly used two parameter probability density function (*pdf*) is the Weibull density function and its two parameters are shape (k) and scale (c). In this study, method of least square (MLE) is used to determine these parameters (Chang, 2011).

A Gaussian mixture model (GMM) is suitable in cases where data to be modeled belong to different data groups. These data sets might be different from each other but number of data points within a group should be the same. Therefore, in statistical sense, a Gaussian Mixture Model (GMM) (Aries et al, 2018) is a probabilistic distribution function which is a combination of many normally distributed data distribution. It is a parametric probability density function evaluated by a weighted sum of individual Gaussian component densities. The use of GMM is motivated by the fact that the data to be fitted looks multimodal. A GMM with weighted sum of M Gaussian component densities is given by the equation:

$$p(x|\lambda) = \sum_{i=1}^M w_i g(x|\mu_i, \Sigma_i) \quad (2)$$

where x is D-dimensional continuous-valued input data vector, that is a measured wind speed and wind direction 2-dimensional input vector, w_i ($i = 1 \dots M$) are weight mixtures and $g(x|\mu_i, \Sigma_i)$ are densities of Gaussian components.

In the presence of training vectors and a GMM configuration, GMM parameters λ , can be estimated in such a way that best describes the training feature vectors distribution. Several techniques are available for estimating GMM parameters (McLachlan and Basford, 1988). The most commonly used technique is Expectation-Maximization (EM) approach which is a two-step procedure i.e. Expectation or E-Step and Maximization or M-Step and uses maximum likelihood (ML) estimation method. In ML approach, model parameters are estimated in such a way which maximizes the likelihood of GMM using a given training data set.

Error Estimation

Error estimations for the fitted functions are performed using Kolmogorov Smirnov (K-S) and coefficient of determination (R^2) tests (Chang, 2011; Abbes and Belhadj, 2012).

Wind Direction and Wind Speed Statistics

Hourly wind speed and directional data is obtained for station number 225 ($52^\circ 27' 46.45''\text{N}$, $4^\circ 33' 19.57''\text{E}$) off the coast of IJmuiden, Dutch province, north Holland (KNMI, Data 2017). These two weather parameters were recorded at 18.5 m height and for the period of 54 years starting from April 01, 1952 to January 01, 2006 (Ohunakin and Akinnawonu, 2012).

Tables 1-3 show the descriptive statistics for wind speeds, wind directions and wind rose data of 54 years period. The entire hourly mean wind speed and direction data is separated into monthly segments and descriptive statistics for each month is determined. Additionally, the statistics is also obtained using entire data set giving a mean value of wind speed as 6.91 m/s with ' σ ' of 3.46 m/s at IJmuiden. The annual mean wind speed values from 1952 to 2006 are shown in Figure 1. Maximum annual mean wind speed (7.73 m/s) is for the year 1954 whereas minimum (5.11 m/s) is for the year 1983. Higher wind speeds are observed in winter time (maximum of 7.93 m/s in January) and lower in summer months (minimum of 6.43 m/s in June) over entire data collection period (Table 1). Figure 2 shows the frequency distribution of wind direction over entire data collection period in different direction bins of 10 degrees. The frequency spread of the hourly wind direction data is also fitted to GMM (Fig. 2). Wind rose diagram is obtained based on hourly mean wind speed and direction values over entire data collection period (Fig. 3). The wind rose indicates that wind blows predominantly from south-west direction and most of wind speed data ranges lie in the fourth quadrant.

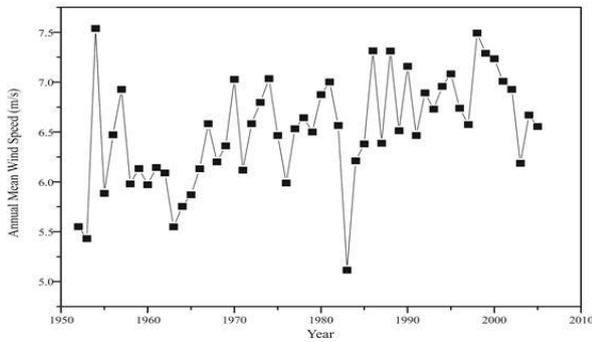


Fig. 1. Yearly averaged wind speed variations over complete 54 years period

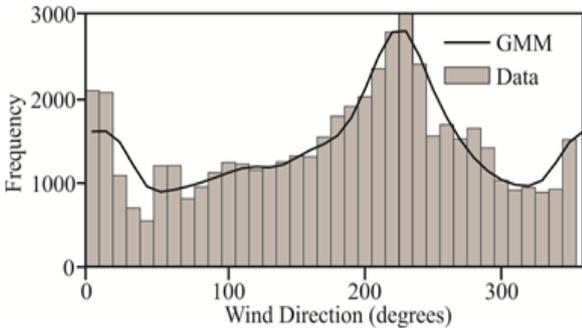


Fig. 2. Histogram of measured hourly averaged wind direction data fitted to GMM

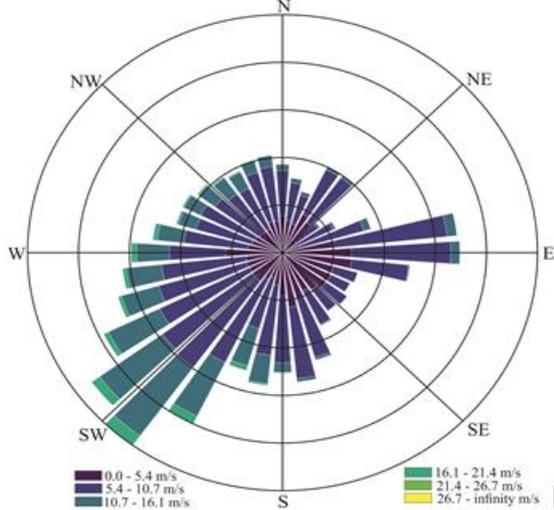


Fig. 3 Hourly averaged wind rose over entire data measurement period

Skewness is the measure of asymmetry of distribution about mean and for the present data set, it is greater than zero though the distribution is moderately skewed but has a slight right tail. The shape of the distribution is described by kurtosis (K) and skewness (S). In January, the kurtosis is less than zero ($K = -0.02$) which implies smaller variance in observed wind speed values with broad peak and smaller tail. The skewness value of $S = 0.56$ implies that the distribution is symmetric (Fig. 4). In the month of June, the kurtosis is slightly greater than zero ($K = 0.15$) implying a comparatively less broad peak and skewness value of $S = 0.54$ indicating a slight tails in positive direction (Fig. 9). For the month of January using 95% confidence level (CL), a confidence interval (CI) of 0.038 and coefficient of variations (CV) of 47.54% are obtained. These results suggest that almost 52% of

total data points are fit into an interval of 0.038 CI centered at mean wind speed (Table 1). Similarly, for the month of June almost 57% of total wind speed points are spread around average wind speed in an interval of 0.028 CI (Table 1). Monthly and overall average wind power densities are also computed (Table 1). It is evident that higher values with maximum of 306.29 W/m^2 in January and lower in summer with a corresponding minimum of 163.43 W/m^2 are observed in June. Wind direction data is moderately skewed negatively during all months except from April to June (Table 2).

The measure of kurtosis (K) predicts the shape of data distribution and its values for all three distributions are much less than 3 (Tables 1 to 3). This indicates that there are fewer outliers compared to a normal distribution. Although kurtosis values for wind speed data distribution are much smaller than 3 but are greater than those for wind direction (Tables 1 and 2). This implies that wind speed distribution is more like a normal distribution. At 95% CL, the CI obtained for overall averaged wind speed distribution is 0.01 and CV of 50.12%. This means that almost 50% of total data points cluster around mean speed in an interval of 0.01 CI. In case of entire data set of wind direction (Table 2), the CI value at 95% confidence level is obtained as 0.28 and CV value of 56.28%. Therefore almost 44% of total wind directions data points fall around mean wind direction in an interval of 0.28 CI. For the entire wind speed data set, wind power density values are determined month-wise using hourly averaged measured wind speed data (Table 1). The wind power density varies between 163.43 W/m^2 and 306.29 W/m^2 , corresponding to the months of June and January respectively. In case of entire data set mean, wind power density is found to be 201.77 W/m^2 . It is evident that higher values are observed in winter (October to March) and lower in summer (April to September).

Table 1 Wind speeds descriptive statistics measured at 18.5 m height (1952–2005).

Month	# of Points	Mean V_{avg} m/s	σ m/s	K	S	P_A W/m^2	CV %	C.I. 95.0%
January	38573	7.93	3.77	-0.02	0.56	306.29	47.54	0.038
February	35000	7.44	3.55	0.28	0.70	252.28	47.72	0.037
March	37786	7.20	3.30	0.22	0.63	229.40	45.90	0.033
April	36201	6.64	2.92	0.06	0.52	179.46	44.00	0.030
May	38658	6.51	2.79	0.29	0.54	169.75	42.91	0.028
June	37330	6.43	2.74	0.15	0.54	163.43	42.63	0.028
July	38589	6.55	2.85	0.27	0.59	172.85	43.46	0.028
August	38268	6.49	2.94	0.31	0.71	168.03	45.32	0.030
September	36608	6.79	3.34	0.27	0.77	191.87	49.23	0.034
October	37790	7.31	3.60	0.35	0.75	239.30	49.35	0.036
November	37113	7.74	3.70	-0.16	0.56	284.78	47.78	0.038
December	38353	7.89	3.71	-0.18	0.51	301.76	47.03	0.037
Overall	461840	$\frac{6.91}{6.90635}$	3.46	0.31	0.60	201.77	50.12	$\frac{0.010}{v}$

Results and Discussion

Weibull distribution function (Carrillo et al, 2014) and Gaussian Mixture Model (GMM) functions are compared using long-term wind speed and direction measured over a period of 54 years from 1952 to 2005. Monthly and overall averages are computed for hourly averaged wind speed and wind direction and fitted to Weibull function and Gaussian Mixture Model (GMM) and pdf are calculated. Using MLE, Weibull function parameters are determined for both monthly and yearly domains. Weibull function is used to determine most probable (V_{mp}), mean (V_m) wind speeds, and power density. Monthly and overall averaged wind power densities are determined using GMM (Table 4). Coefficient of determination (R^2) and Kolmogorov Smirnov (K-S) are performed as goodness-of-fit tests to check the authenticity of Weibull function and GMM (Table 4). Monthly and overall histograms and probability density curves are plotted (Figs. 4, Fig. 16).

Table 2 Wind direction descriptive statistics measured at 18.5 m height (1952–2005).

Month	# of Points	Mean direction	σ m/s	K	S	CV %	C.I. 95.0%
January	38573	196.61	97.94	-0.66	-0.57	49.81	0.98
February	35000	175.23	106.33	-1.15	-0.24	60.68	1.11
March	37786	173.44	103.14	-1.06	-0.21	59.46	1.04
April	36201	153.67	98.67	-1.06	0.18	64.21	1.02
May	38658	151.07	97.85	-1.01	0.17	64.77	0.99
June	37330	158.81	86.88	-0.76	0.01	54.71	0.88
July	38589	163.92	83.42	-0.48	-0.08	50.89	0.83
August	38268	165.33	92.53	-0.74	-0.05	55.97	0.93
September	36608	180.89	99.00	-0.93	-0.22	54.73	1.01
October	37790	195.78	102.60	-0.81	-0.48	52.41	1.03
November	37113	198.59	98.64	-0.74	-0.52	49.67	1.00
December	38353	192.96	100.31	-0.76	-0.50	51.99	1.00
Overall	461840	175.58	98.81	-0.97	-0.19	56.28	0.28

Table 3 Wind rose descriptive statistics with speeds measured at 18.5 m height 1952–2005).

Month	# of Points	Mean direction	σ m/s	K	S	CV %	C.I. 95.0%
January	38573	184.12	84.39	-0.69	-0.24	45.83	0.84
February	35000	172.35	91.88	-1.05	-0.04	53.31	0.96
March	37786	178.29	94.61	-1.05	-0.13	53.06	0.95
April	36201	173.25	110.16	-1.31	-0.02	63.58	1.14
May	38658	176.87	109.91	-1.31	-0.07	62.14	1.11
June	37330	195.23	108.30	-1.09	-0.42	55.47	1.10
July	38589	207.08	102.30	-0.80	-0.63	49.40	1.02
August	38268	196.44	102.16	-1.07	-0.40	52.01	1.02
September	36608	186.81	95.96	-1.02	-0.22	51.37	0.98
October	37790	182.20	86.65	-0.86	-0.10	47.56	0.87
November	37113	184.40	86.71	-0.74	-0.16	47.02	0.88
December	38353	183.77	86.19	-0.79	-0.19	46.90	0.86
Overall	461840	184.74	97.58	-1.03	-0.22	52.82	0.28

Goodness-of-fit R^2 and K-S tests values for Weibull function and GMM indicate that GMM is a better choice in both for monthly distribution of hourly averaged and overall wind speed data (Figs. 4-15). Although R^2 values obtained for Weibull function are greater than R^2 values obtained for GMM for all months and for entire wind speed data sets, nevertheless based on K-S test statistics alone, GMM proved superior fit function compared to Weibull function. This is because R^2 statistics gives a measure of variability in the modeled function against the measured or recorded data distribution. The higher R^2 values imply that the approximated function follow the variations present in the measured data set. K-S test on the other hand not only describes variability, additionally it also tests the equality of two distributions. Thus, based on K-S test statistics results the measured distribution is better approximated by GMM. Specifically, except for the months of January, November and December, for all other months smaller K-S test statistics values suggest that GMM outperforms Weibull function. Furthermore, in the present investigation wind power density values obtained using GMM are much closer to the power density values obtained directly from measured wind speeds. This can be explained by the fact that since power density varies as cube of wind speeds and so is heavily weighted on wind speeds. For overall hourly averaged wind speed data for the entire data collection period, the GMM appears to be a better fit to the measured data (Figs. 16, 17). As far as wind power density estimation is concerned, it is evident from these figures that GMM based distribution overestimated the frequency during January to March (Figs. 4-6) and October to December (Figs. 13-15) while underestimated from April to August (Figs. 7-11). However, in the month of September the two distributions behaved more or less the same (Fig. 12).

Table 4 Weibull and GMM wind characteristics and wind power densities

Month	Weibull Density Function							Gaussian Mixture Model (GMM)		
	k	c m/s	V_m m/s	V_{mp} m/s	P_w W/m ²	R^2	K-S	R^2	K-S	P_G W/m ²
January	1.97	8.06	7.14	3.98	432.67	0.995	0.077	0.917	0.107	284
February	1.93	7.43	6.58	3.59	346.20	0.995	0.092	0.980	0.047	273
March	2.04	7.30	6.47	3.73	310.05	0.994	0.089	0.983	0.039	231
April	2.13	6.70	5.93	3.57	229.71	0.997	0.088	0.974	0.059	295
May	2.19	6.58	5.82	3.58	212.20	0.998	0.095	0.957	0.081	231
June	2.17	6.40	5.67	3.46	197.43	0.998	0.104	0.946	0.086	184
July	2.11	6.49	5.75	3.43	210.83	0.997	0.113	0.959	0.074	193
August	1.98	6.31	5.59	3.13	206.52	0.991	0.122	0.953	0.067	212
September	1.82	6.61	5.87	2.99	261.51	0.985	0.112	0.970	0.037	202
October	1.85	7.23	6.42	3.32	337.25	0.991	0.099	0.988	0.030	307
November	1.94	7.79	6.91	3.79	396.20	0.988	0.080	0.946	0.081	262
December	2.02	8.10	7.18	4.10	427.97	0.992	0.074	0.904	0.104	221
Overall	2.19	8.15	7.22	4.43	403.96	0.921	0.046	0.989	0.015	307

Figs. 18 and 19 give the comparison between R^2 and K-S test values for the two functions that is, GMM and Weibull functions fittings to the overall data set.

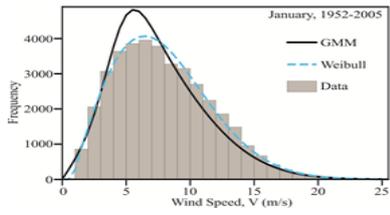


Fig. 4. Monthly histogram of pdf function for January

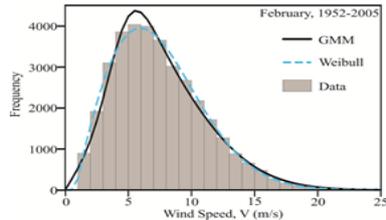


Fig. 5. Monthly histogram of pdf function for February

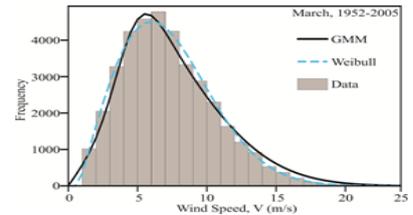


Fig. 6. Monthly histogram of pdf function for March

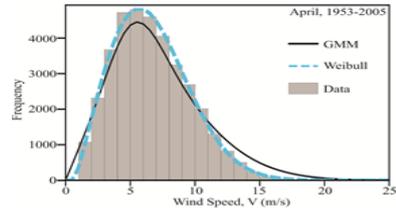


Fig. 7. Monthly histogram of pdf function for April

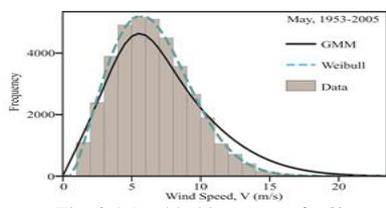


Fig. 8. Monthly histogram of pdf function for May

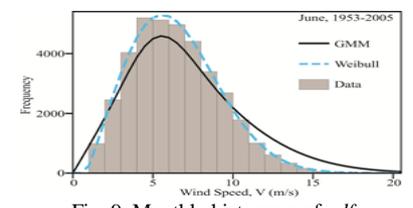


Fig. 9. Monthly histogram of pdf function for June

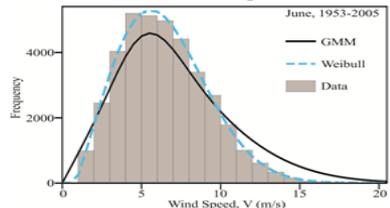


Fig. 10. Monthly histogram of pdf function for July

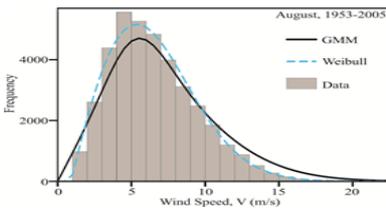


Fig. 11. Monthly histogram of pdf function for August

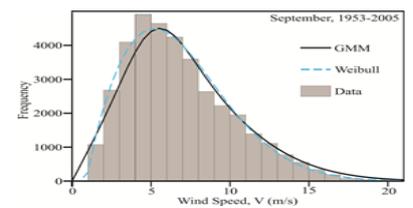


Fig. 12. Monthly histogram of pdf function for September

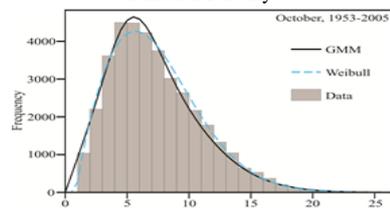


Fig. 13. Monthly histogram of pdf function for October

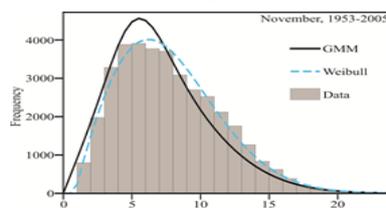


Fig. 14. Monthly histogram of pdf function for November

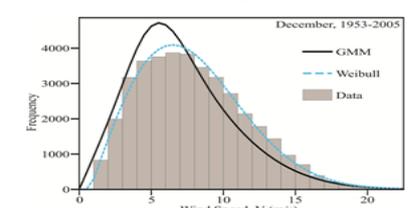


Fig. 15. Monthly histogram of pdf function for December

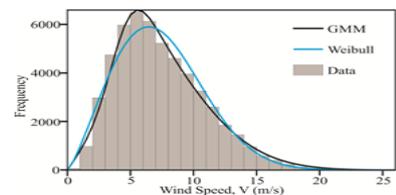


Fig. 16. Comparison between measured wind speed and probability density functions for the entire data set.

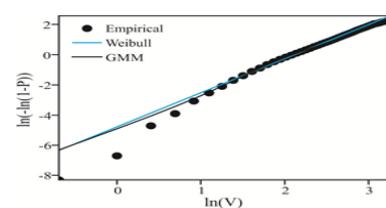


Fig. 17. Empirical method for fitting wind speed

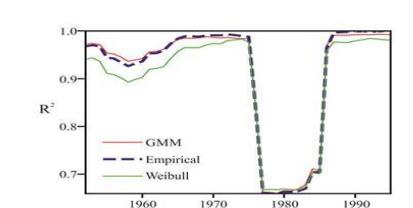


Fig. 18. R² traces for fitted functions

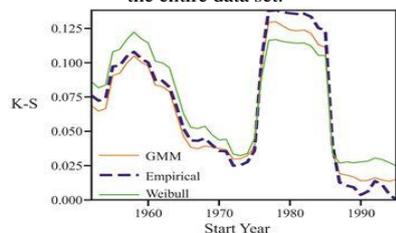


Fig. 19. K-S test traces for fitted functions

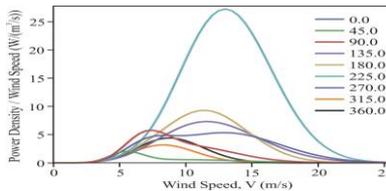


Fig. 20. Variation in wind power density values with wind speeds for some selected wind directions.

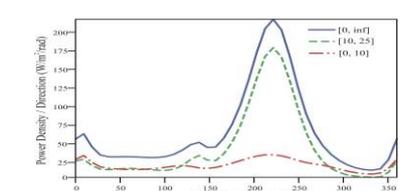


Fig. 21. Variability of wind power density versus wind direction

Figures show superior performance of GMM function compared to Weibull function. Specifically, during February to October GMM performed better compared to Weibull functions. However, from November to January, the Weibull function performed slightly better than GMM (R^2 and K-S test statistics for Weibull and GMM in Table 4). Monthly wind power densities and

wind power density for the entire data set are determined using Weibull and GMM functions. Wind power densities (P_G) obtained using GMM for monthly and overall wind data set are closer to the values determined using actual wind speed data (P_A), listed in Table 1, column 7 and Table 4, column 11. Weibull function gave overestimation of wind power density

for all months which is approximately 10 to 30 percent, while GMM method underestimating from November to December. In the remaining months, power density values determined are slightly overestimated.

Wind energy potential estimated for a site is sensitive to variations in wind speeds and direction of wind speeds. Figures 20 and 21 are traces of wind power density plotted against wind speed and wind direction, respectively. The data revealed that most of the wind speed ranging between 5 m/s to 20 m/s is blowing in from the south-west direction (Fig. 20). From south-west direction, majority of data points are in the vicinity of 13 m/s with corresponding power density greater than 25 W/m². The wind power density distribution from south-west direction is symmetrical around 13 m/s.

Furthermore, wind power density distribution as a function of wind direction for three ranges of velocities i.e. 0 - infinity, 10 - 25, and 0 - 10 m/s (Fig. 21). The peak of wind power distribution for velocity range 0 to infinity occurs at around 225 degrees i.e. from south-west direction. Furthermore, the area under the curve for wind speeds ranging from 0 to infinity is the largest, i.e. possibility of maximum wind energy extraction. Next best wind power density is obtained for wind speeds ranging between 10 to 25 m/s. The lowest wind power density values are observed for wind speeds ranging between 0 to 10 m/s, suggesting higher density of data points in the range 10 to 25 m/s. In all three distributions, peak is observed at 225 degrees i.e. from south-west direction.

Conclusion

In the present paper, authors have assessed the greater suitability of Weibull or Gaussian Mixture Model (GMM) as fitting functions to wind speed and directional data. A high density data set of 54 years duration and consisting of 461840 hourly wind speed and directional data points are used. The long term data set has monthly wind speeds ranging between 6.43 m/s to 7.93 m/s. For the complete data set wind power density is 202 W/m² and yearly values are in the range 163 W/m² and 306 W/m².

In case of monthly average wind speed distribution, Weibull show good performance and appears to be a better choice over GMM in approximating measured wind speed data distribution. This behavior is consistent not only in terms of observed Weibull profile but also from calculated R² values (Table 4) for Weibull function. However, as regards the equality of two functions, that is, Weibull and GMM, to the measured data distribution, GMM has the higher precedence. This is evident from the K-S test statistics values (Table 4) which are smaller for GMM than Weibull function. Nevertheless, for the complete data set GMM performed better than Weibull function in

terms of fitting of the measured data distribution. Contrary to the above conclusion, monthly wind power density values for majority of months obtained using GMM are closer to power density values obtained directly from measured data. This observation is also true for the overall data set as well. The peak of the wind power density is found to be occurring around 225° wind direction. The use of long term wind speed and direction data give a reliable estimate of wind power potential which is helpful in the design of wind farm in the targeted area.

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